Periodic Heat Conduction Through Composite Panels

Lit S. Han*
Ohio State University, Columbus, Ohio

I. Introduction

PROBLEMS in unsteady conduction through layered media are traditionally solved by Green's functions, a transform technique, or the eigenvalue approach. These methods are amply described in treatises by, for example, Carslaw and Jaeger[1] and, more recently, Mikhailov et al.,[2] and they entail varying degrees of complexity, depending on the problem specification and, in particular, on the number of layers involved. The latter specification dictates the manipulative labor required in matching individual layer solutions through their interfacial boundary conditions. For composite panels comprising a large number of laminates, the classical methods can be quite forbidding, and even numerical methods may be burdensome. Alternatively, continuum models are being developed with the aim of mimicking the thermal responses of composites by using single effective thermal properties reflective of the physical makeups of the layered media. Indeed, the theory of modeling is such a challenge that it has attracted the attention of many a well-known researcher. Significantly, the work of Maewal et al.[3] contributed to the criterion of various orders of approximation based on the properties and dimensions of the laminates; Schimmel and his colleagues[4] emphasized the effect of the stacking order of the laminates. Essential in developing a continuum model are exact mathematical solutions for the purpose of conceiving and validating a model. Exact solutions are, therefore, not only useful in their own right but also invaluable tin revealing parametric interactions in the model under consideration.

In this Note we describe a simple but exact methodology of analyzing linear, one-dimensional, periodic heating-cooling problems of composites with any number of laminates. It is a numeric-analytical approach using complex algebra and calculating two complex coefficients from one layer to the next.

Shown in Fig. 1 is a composite panel with n layers of different, isotropic material of which the jth layer has constant physical properties ρ_j, C_j, and k_j and thickness L_j. A contact resistance V_j governs the heat flow at the left interface. For convenience in algebraic operations, each layer has its own axis with origin at the left-hand face, extending from $x_j = 0$ to $x_j = L_j$. At the latter position, a new origin $x_{j+1} = 0$ is defined for the next layer, and so on. Expressed in this fashion, each layer is governed by

$$(\partial^2 T_j/\partial x_j^2) = (\partial T_j/\partial \theta)/\alpha_j \qquad j = 1 \ldots n \tag{1}$$

Without loss of generality, the left-hand, exposed surface of Fig. 1, $x_1 = 0$, may be specified to have a fixed temperature or zero heat flux; and the right-hand, exposed face $x_n = L_n$ is to have one of the following periodic boundary conditions:

$$T_n = T_e \exp(i\omega\theta) \tag{2a}$$

$$k_n(\partial T_n/\partial x_n) = q_e \exp(i\omega\theta) \tag{2b}$$

$$k_n(\partial T_n/\partial x_n) = h[T_e \exp(i\omega\theta) - T_n] \tag{2c}$$

where the real part of the exponential function is inferred and T_e, q_e, and h are a reference temperature, heat flux, and heat-transfer coefficient, respectively.

II. Solution Methodology

The boundary conditions of Eqs. (2) with a frequency ω dictate that each layer has a solution of the same frequency given by

$$T_j = [A_j \cosh(\beta_j x_j) + B_j \sinh(\beta_j x_j)] \exp(i\omega\theta) \tag{3}$$

in which the eigencoefficient β_j is related to ω and α_j by

$$\beta_j = (1+i)\sqrt{\omega/2\alpha_j} \tag{4}$$

In Eq. (3), coefficients A_j and B_j are naturally complex as is β_j of Eq. (4).

The boundary condition on the left-hand face, $x_1 = 0$, may be satisfied by letting $A_1 = 0$ for a zero surface temperature, or $B_1 = 0$ for the zero flux condition. For definiteness, the latter condition is taken, i.e., $B_1 = 0$ but A_1 is left undefined.

At the interface beteen two adjacent layers j and $(j+1)$, equality of the heat fluxes results in a simple expression relating B_{j+1} with the solution coefficients A_j and B_j,

$$B_{j+1} = \sqrt{(k\rho C)_j/(k\rho C)_{j+1}}\,[A_j \sinh(\beta_j L_j) + B_j \cosh(\beta_j L_j)] \tag{5}$$

The interface temperatures of these two layers differ by an amount depending on the contact resistance V_{j+1}, resulting in

$$A_{j+1} = A_j \cosh(\beta_j L_j) + B_j \sinh(\beta_j L_j) + (V_{j+1}k_{j+1}/L_{j+1})(\beta_{j+1}L_{j+1})B_{j+1} \tag{6}$$

Equations (5) and (6) are a set of recurrence formulas that calculate the complex coefficients from one layer to the next. The process, beginning with $j = 1$, goes on until the last layer n is reached, at which point the boundary condition at $x_n = L_n$, Eq. (2), is invoked.

Noting that A_1 is undefined but is carried along in the marching process, the recurrence relations for A_j and B_j, Eqs. (5) and (6), may be interpreted as those for their ratios to A_1.

Defining $\bar{A}_j = A_j/A_1$ and $\bar{B}_j = B_j/A_1$, Eqs. (5) and (6) remain unchanged but are understood to be in barred notations for the coefficients. For example, the second-layer coefficients $\bar{A}_2$ and $\bar{B}_2$ are

$$\bar{B}_2 = \sqrt{(k\rho C)_1/(k\rho C)_2}\,[\sinh(\beta_1 L_1)] \tag{7}$$

$$\bar{A}_2 = \cosh(\beta_1 L_1) + (V_2 k_2/L_2)(\beta_2 L_2)B_2 \tag{8}$$

Using the recurrence relations, Eqs. (5) and (6), in barred notations, all coefficient ratios $\bar{A}_j$ and $\bar{B}_j$ from $j = 2$ to n are obtained. At the last layer n, the three boundary conditions at $x_n = L_n$ of Eq. (2) produce the following expressions for A_1:

$$A_1 = T_e/[\bar{A}_n \cosh(\beta_n L_n) + \bar{B}_n \sinh(\beta_n L_n)] \tag{9a}$$

$$A_1 = (q_e/k_n\beta_n)/[\bar{A}_n \sinh(\beta_n L_n) + \bar{B}_n \cosh(\beta_n L_n)] \tag{9b}$$

$$A_1 = T_e/\{(k_n/L_n h)(\beta_n L_n)[\bar{A}_n \sinh(\beta_n L_n) + \bar{B}_n \cosh(\beta_n L_n)] + [\bar{A}_n \cosh(\beta_n L_n) + \bar{B}_n \sinh(\beta_n L_n)]\} \tag{9c}$$

Recalculating the coefficients by multiplying the coefficient ratios by A_1 completes the analysis. Numerically the entire

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*Professor of Mechanical Engineering.

process can be easily carried out by a simple computer routine in which the number of layers is of little concern. For each layer, the temperature distribution in Eq. (3) can be expressed by two component parts: $R_j(x_j)$ in phase with the frequency ω and $I_j(x_j)$, out of phase. More specifically, the layer temperature is now

$$T_j = T_e[R_j \cos(\omega\theta) - I_j \sin(\omega\theta)] \tag{10}$$

For the first boundary condition of Eq. (2), it is required $R_n(L_n) = 1$ and $I_n(L_n) = 0$; for the other two conditions, terminal requirements such as those can be established to check numerical accuracy.

III. Example: A Four-Layer Composite

As a demonstration, the numeric-analytical procedure is applied to a composite panel with for layers of equal thickness.

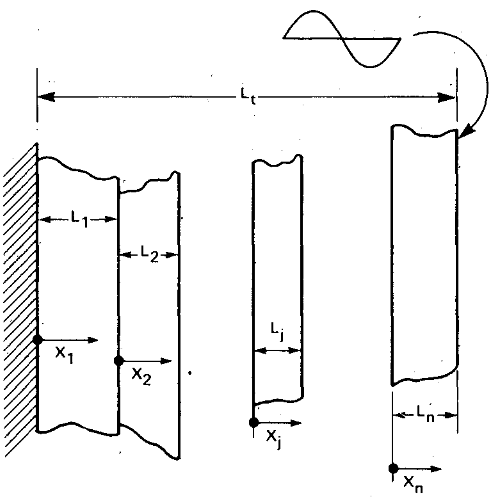

Fig. 1 Panel configuration and definitions.

The boundary condition at $x_1 = 0$ is zero heat flux and that at $x_n = L_n$ is a fluctuating temperature, i.e., condition (2a) with $T_e = 1$.

For convenience, although it is not necessary, a reference time scale is derived from the minimum diffusivity of layers α_m and their overall thickness L_t, equal to $4L_1$ in this example. Nondimensional time and frequency are therefore indicated by

$$\bar{\omega} = \omega(L_t^2/\alpha_m), \qquad \bar{\theta} = \theta(\alpha_m/L_t^2) \tag{11}$$

satisfying the obvious requirement $\omega\theta = \bar{\omega}\bar{\theta}$. With these definitions, the eigencoefficients $\beta_j L_j$ can be expressed in dimensionless ratios as

$$\beta_j L_j = (1+i)(L_j/L_j)\sqrt{\bar{\omega}/2}\sqrt{\alpha_m/\alpha_j} \tag{12}$$

And by fixing a value for $\bar{\omega}$, the coefficient-ratios $\bar{A}_{j+1}$ and $\bar{B}_{j+1}$ are computed from Eqs. (5) and (6), where barred notations are used to replace the unbarred ones. The range of $\bar{\omega}$ extends from 4 to 256. Computed temperature profiles expressed in terms of R_j and I_j of Eq. (10) are shown in Figs. 2a and 2b. The thermal properties of the four layers have K and (ρC) in the ratios 1:2:4:8, starting from the inner layer for the data in Fig. 2a, and a reverse stacking order for Fig. 2b. In both arrangements the thermal diffusivities are equal for all layers. Examination of the profiles reveals, in accord with Ref. 4, a strong influence of the laminate stacking order at low frequencies for which thermal waves have greater penetration depths. At high frequencies, however, data in these two figures are not revealing enough to support conclusions, for the wave depths are shallower and their amplitudes nearly equal, owing to the equal diffusivity for all layers. Need for modeling at high frequencies based on a time-dependent parametric ordering, first studied in Ref. 3, is thus indicated.

Since the purpose of this Note is not to study modeling parameters extensively but to introduce a simple methodology, we shall conclude by presenting a performance

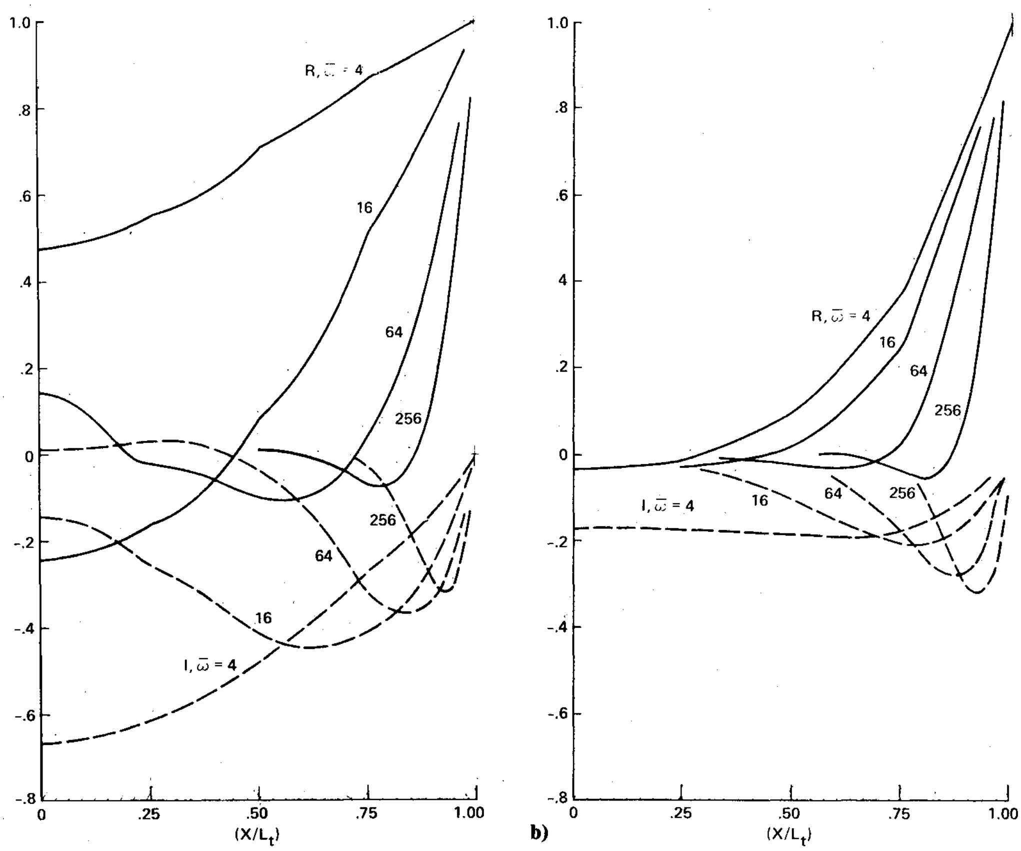

Fig. 2 Temperature distributions in a four-layer composite: a) k, ρC = 1:2:4:8; b) k,ρC = 8:4:2:1.

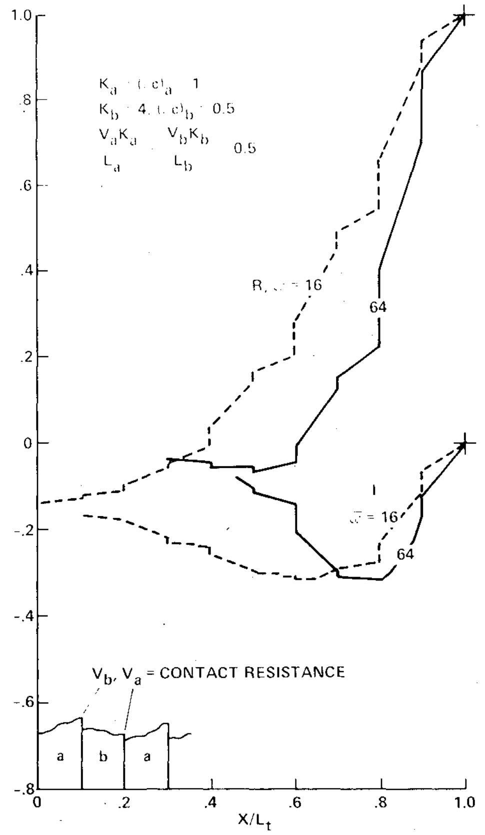

Fig. 3 Temperature profiles in a ten-layer composite.

diagram for a two-material, ten-layer composite with interface contact resistances. Specifications are self-explanatory in Fig. 3. Note the assumed dimensionless value of $(V_j k_j / L_j) = 0.5$, which makes the contact resistances unequal in their physical units, thus causing uneven gaps in the curves.

IV. Composite Spheres and Cylinders

For composite spheres consisting of a number of shells, the method of analysis is essentially unchanged because of similar eigenfunctions in individual sphere-layer solutions. All that is needed is to modify Eq. (2) by dividing the right-hand side by a local radius and interpreting x_j as the local radius minus the inner radius of the jth shell. The resulting recurrence relation remains largely intact. As for composite cylinders, however, the functional structure of Eq. (2) is changed into that of Bessel functions in complex arguments, with some numerical complications that can be dealt with.

Acknowledgment

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Prediction of Film Boiling Wakes Behind Cylinders in Cross Flow

Rajeev Kaul*
Pipeline Hydraulics Engineering Inc.
Houston, Texas
and
Larry C. Witte†
University of Houston, Houston, Texas

Nomenclature

C_N	= cavitation number, see Eq. (3)
D	= cylinder diameter
Fr	= Froude number, see Eq. (5)
g	= gravity
p	= pressure
Δp	= pressure difference
R	= cylinder radius
T	= temperature
ΔT_B	= temperature difference, $T_{sat} - T_B$
u	= velocity in the x direction
V	= cross-flow velocity
W	= complex potential function, $\phi + i\psi$
WL	= wake length
WT	= wake thickness
x	= cylindrical surface coordinate
y	= cylindrical surface normal coordinate
Z	= complex function, $x + iy$
θ	= angle measured from the stagnation point
μ	= viscosity
ρ	= density
δ	= vapor film thickness
ϕ	= velocity potential function
ψ	= stream function

Subscripts

B	= bulk
l	= liquid
obs	= observed
s	= separation point
sat	= saturation
th	= theoretical
v,vap	= vapor
wk	= wake
w	= wall
∞	= freestream

Introduction

FILM boiling is often encountered in cases such as handling of cryogenic fluids, quenching of metal parts, cooling of rocket nozzles, etc. A film of vapor separates the liquid from its heat source during this boiling mode. Film boiling can be classified into two main groups—natural convection (or pool) and forced convection (or flow) film boiling. There is no forced relative motion between the liquid and the hot surface in pool boiling. In contrast, flow boiling involves relative motion between the liquid and the surface.

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*Project Engineer.
†Professor of Mechanical Engineering. Senior Member AIAA.